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VIBRATION CONTROL IN MACHINES AND STRUCTURES USING VISCOELASTIC DAMPING

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The work on vibration control of machines and structures incorporating viscoelastic materials in suitable arrangements, is highlighted for situations involving vibration excitations over a broad frequency range. The principle involved is that of vibratory energy dissipation due to damping as a result of deformation of viscoelastic materials. The characteristics of viscoelastic materials have to be suitably modelled in view of their dependence on several factors like type of dynamic excitation, temperature, strain, etc. The main objective of the paper is to review some of the salient work, done at I.I.T. Delhi, relating to constrained layer damping for structures, optimisation studies and support damping for rotor systems. Some results, which have either not been published or are not easily available, are also included. Brief reference to related work, carried out at other places, is also made, the list being by no means exhaustive, due to space constraints. Efforts have been made to predict the future trends in theoretical and experimental work in the analysis, optimisation and use of viscoelastically damped structures.

1. INTRODUCTION

The use of viscoelastic damping is beneficial in situations involving a wide range of excitation frequencies. There are two types of basic configurations as in Figure 1 where the vibratory energy is dissipated due to direct strains in the case of unconstrained viscoelastic materials bonded to the elastic layer and predominantly shear strains in the constrained viscoelastic materials. Some of the high polymers are known to exhibit viscoelastic behaviour.

The structural elements to which additive damping is applied, may be in the form of beams, plates, rings, shells etc. Analysis of such elements subjected to vibratory excitations, have been carried out since the early sixties assuming viscoelastic materials (VEMs) to be linear. For any mode n, the system loss factor η_s is obtained from the response solution equations like

$$[-m\omega^2 + k(1 + \mathrm{i}\eta_s)]q_n = f_n, \tag{1}$$

where f_n is the generalised excitation and q_n, the corresponding displacement. In the above, m and k refer to the generalised mass and stiffness respectively. In much of the work, the variation of η_s or $k\eta_s$ is reported for various geometrical and physical parameters. A review of the work is reported in references [1–3]. The VEMs may be used in the form of discrete dampers, absorbers or, alternatively, the surface of the structural member may be partially covered instead of being fully covered. There have been applications in aerospace,

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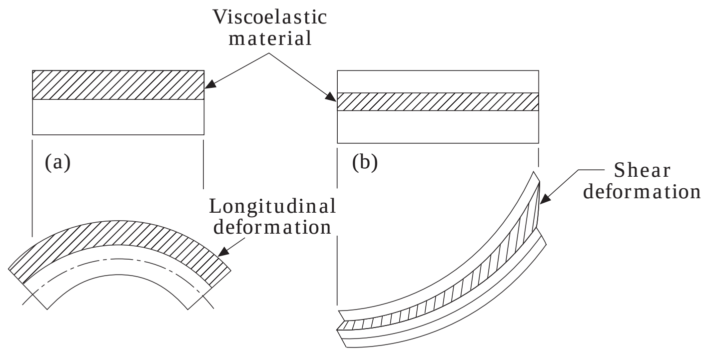

Figure 1. Basic configurations for viscoelastic damping. (a) Unconstrained and (b) constrained treatment.

automotive, machine tools, ships, turbines, electronic and optical equipment etc., [4–7]. These are essentially passive dampers and a number of developments have been reported at "damping" conferences, periodically organised since the conference in 1978 on "damping technology in the 1980s" at Dayton, Ohio. There has also been considerable work in the active control of vibrations and it has been felt that an optimal blend of passive damping and active control is useful. The present paper deals with developments and future trends in analysis and optimisation of viscoelastically damped structures. Some of the work pertaining to constrained layer damping treatment for structures, optimisation and analysis of rotor systems with support damping, carried out at I.I.T. Delhi, has been reviewed with brief reference to the related work in other places. A comprehensive review is not attempted, due to space constraints and the fact that it is available in other texts [3, 4].

2. VISCOELASTIC MATERIAL CHARACTERISATION

If a viscoelastic material (VEM) becomes strained due to harmonic stress, the strain is not in phase but lags behind by an angle θ, which is a measure of the damping in the material. A common method of representation of damping is by the loss factor η of the material which equals $\tan\theta$. η is also equal to the ratio of energy dissipated to that stored in the material. The ratio of stress to strain in a VEM, under harmonic excitation conditions is represented by complex moduli $E(1 + i\beta)$ and $G(1 + i\eta)$ in direct and shear strain, respectively. These properties are seen to depend on frequency, temperature and strain. The plots of in-phase shear modulus G and loss factor η in shear for plasticized P.V.C. are shown in Figure 2. In Figure 2(a), these are plotted against frequency f for varying temperatures while in Figure 2(b), the plots are for different values of shear strain amplitude. These were measured in direct shear of the viscoelastic specimen with the dynamic force applied by an electro–dynamic vibrator, for varying frequencies. The shear strain amplitude was varied at different frequencies by the power amplifier of the vibrator.

The temperature–frequency superposition principle [8] forms the basis of reduction of the three dimensional relation between the in-phase modulus (or loss factor), frequency and temperature to a two dimensional one. This involves the use of the reduced frequency or the reduced temperature, which combines the effects of frequency and temperature by the use of factors known as shift factors. These factors are often found empirically [4, 9].

Since the complex modulus representation is applicable for harmonic vibrations only, a different representation is needed for transient and other types of excitations. One of the techniques uses the differential operator form of the relationship between stress and strain,

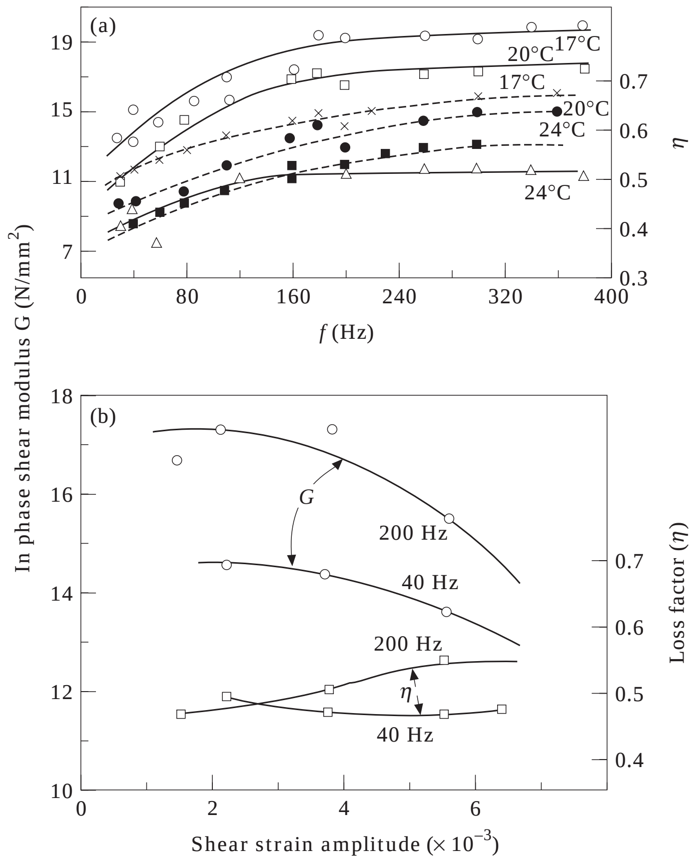

Figure 2. Dynamic properties of plasticized P.V.C. (a) η for varying temperature as function of frequency, (b) G and η as function of shear strain at 20.5° temperature. Dashed lines in (a) refer to loss factor (η).

the constants of which are found from the plots of complex modulus against frequency, for a given temperature [10]. One of the recent developments is to use the fractional calculus approach for modelling the behaviour of viscoelastic materials. This involves use of derivatives of fractional order and is reported to represent accurately the observed dynamic properties over a wide range of frequencies [11–13]. These are seen to be extensions of the classical exponential models of viscoelastic relaxation [14] and have been applied in a limited number of cases of viscoelastically damped structural elements.

3. ANALYSIS

For damping and response analysis of viscoelastically damped structural elements, the governing equations need to be formulated and solved. For harmonic excitation, the correspondence principle of viscoelasticity is used whereby the solution for viscoelastically damped cases can be obtained from that of the corresponding elastic one, by using complex moduli in place of elastic ones. For complex structures, the work on the finite element method (FEM) has been referred to, in addition to that on structural dynamic modification (SDM), or reanalysis. For non-harmonic excitations, an operator form of the stress–strain law has been used in section 3.4.

3.1. VISCOELASTICALLY DAMPED BEAMS WITH HARMONIC EXCITATION

Figure 3 shows the constrained type beam with an assumed displacement variation across the thickness. It is assumed that layers 1 and 3 bend as Bernoulli–Euler beams with $\bar{u}_1 = \bar{u}_3 = w'$, w being transverse displacement and $'$, the differentiation with respect to x for shear in layers 1 and 3, is ignored; w is assumed constant at a section. Both shear and direct strains are accounted for in core 2 and the material of core 2 is assumed as linear or strain independent and no slip is assumed at the interfaces.

The shear angle γ_2 in the core is

$$\bar{\alpha} - w' = (u_1 - u_3)/t_2 - (w'/t_2)a, \tag{2}$$

where t_i is the thickness of layer i ($i = 1, 2, 3$), $a = t_2 + (t_1 + t_3)/2$ and u_i is the longitudinal displacement of the middle of layer i ($i = 1, 3$).

The expression for strain energy U of the beam, assuming all layers as elastic, is given by

$$U = \int_0^L \Bigg[s_2\,\gamma_2^2 + r_1\,u_1'^2 + r_3\,u_3'^2 + (q_1 + q_3)\,(w'')^2 + c_2 \bigg\{ (u_1'^2 + u_3'^2 + u_1'\,u_3' + \frac{(w'')^2}{4}(t_1^2 + t_3^2 - t_1\,t_3) + u_1'\,w'' \left(\frac{t_3}{2} - t_1 \right) + u_3'\,w'' \left(t_3 - \frac{t_1}{2} \right) \bigg\} \Bigg] \mathrm{d}x, \tag{3}$$

where $s = \frac{1}{2}\,bG_2\,t_2\,k_2$, $q_i = (b/24)E_i\,t_i^3$, $r_i = (b/2)E_i\,t_i$, $c_2 = bE_2\,t_2/6$, L = length, b = width, E_i = Young's modulus of layer i, G_2 = shear modulus of layer 2, k_2 = shear coefficient. The kinetic energy T due to longitudinal, rotational and transverse displacements is

$$T = \int_0^L \Bigg[\frac{\rho}{2}(\dot{w})^2 + \frac{b\rho_1\,t_1}{2}\,\dot{u}_1^2 + \frac{b\rho_3\,t_3}{2}\,\dot{u}_3^2 + (\dot{w}')^2 \left(\frac{\rho_1\,t_1^3 + \rho_3\,t_3^3}{24} \right) b + \frac{b\rho_2\,t_2}{2} \left(\frac{\dot{u}_1 + \dot{u}_3}{2} + (\dot{w}')^2 \varepsilon_1 \right)^2 + \frac{b\rho_2\,t_2}{24}(\dot{u}_1 - \dot{u}_3\,\dot{w}'\varepsilon_2)^2 \Bigg] \mathrm{d}x, \tag{4}$$

where $\varepsilon_1 = (t_3 - t_1)/4$, $\varepsilon_2 = (t_1 + t_3)/2$, $(\dot{\ })$ is differentiation with respect to time t, ρ_i = mass density of layer i and ρ = mass per unit length.

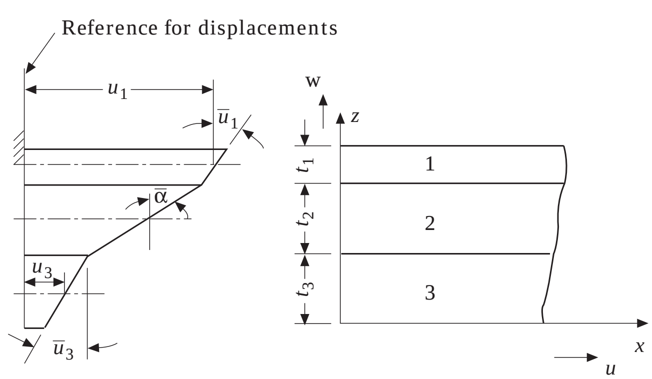

Figure 3. Constrained type viscoelastically damped arrangement.

The energy V due to the excitation of intensity $f(x) \sin \omega t$ is

$$V = \int_0^L f(x) \sin \omega t w \, \mathrm{d}x. \tag{5}$$

Using Hamilton's principle, three governing equations and boundary conditions can be obtained [15, 16]. The solution for simply supported ends is assumed as

$$w = \sum_{n=1}^{\infty} w_n \sin \frac{n\pi x}{L} \sin \omega t, \qquad u_i = \sum_{n=1}^{\infty} u_{in} \cos \frac{n\pi x}{L} \sin \omega t, \qquad (i = 1, 3). \tag{6}$$

Also, $f(x)$ is written as

$$\sum_{n=1}^{\infty} f_n \sin \frac{n\pi x}{L}.$$

Substituting equations (6) in the governing equations and also using $G_2(1 + i\eta_2)$ in place of G_2, one can get equations for displacements, η_s and $k\eta_s$. Three families of modes are obtained for each value of n viz. flexural, extensional and thickness shear [16] for which system loss factors can be found.

If only transverse inertia effects are included and the extensional effect of core 2 is ignored, the governing equations are

$$\frac{b}{12}(E_1 t_1^3 + E_3 t_3^3) w'''' - b t_2 G_2 \frac{a}{t_2}\left(w'' \frac{a}{t_2} - \frac{E_1 t_1 + E_3 t_3}{E_3 t_3} \frac{u_1'}{t_2}\right) + \rho \ddot{w} = f(x) \sin \omega t, \tag{7}$$

$$b G_2 m' \left(\frac{w' a}{t_2} - \frac{u_1 m'}{t_2}\right) + \bar{D} u_1'' = 0, \qquad u_1 = -u_3/m, \tag{8, 9}$$

where

$$m = (E_1 t_1)/(E_3 t_3), \qquad m' = 1 + m, \qquad \bar{D} = b(E_1 t_1 + E_3 t_3 m^2).$$

From the above equations, a sixth order equation in w or u_1 may be obtained which is identical to that in reference [17, 18]. The expression for η_s is also identical to that by Ross *et al*. [19]. η_s is seen to be dependent on shear parameter $\psi_{2,3} = G_2/[E_3 t_3^2 (n\pi/L)^2]$, $\theta_{i,j}$ (where $\theta_{i,j} = t_i/t_j$) and $\alpha_{i,j} = E_i/E_j$, $i = 1, 2$ and $j = 3$. Figure 4 shows the variation of η_s with $\psi_{2,3}$ and $\theta_{2,3}$. It is seen that the value of the system damping loss factor is optimum for only one value of $\psi_{2,3}$ and any change due to change in n viz. modal number or G_2 (changing with frequency or temperature) would change the value of η_s. Efforts have been made to reduce the variation by using multicored systems [20] as in Figure 5 or multilayered arrangements using different viscoelastic materials.

A general analysis, applicable to any number of layers, with alternate elastic and viscoelastic layers, is reported in reference [21] and results for system loss factor for constant weight, size and static stiffness are reported in reference [22]. There is in general an increase in maximum system loss factor with an increase in the number of layers.

Refinements in the analysis continue to be made. If shear effect is considered in layers 1 and 3, five governing equations are obtained as given in Appendix A. The formulation is in terms of displacement u_2 of the middle layer 2, rotations $\bar{u}_1$, $\bar{u}_2$, $\bar{u}_3$ of the sections in layers 1, 2, 3 respectively, and transverse displacement w at a section. A comparison of

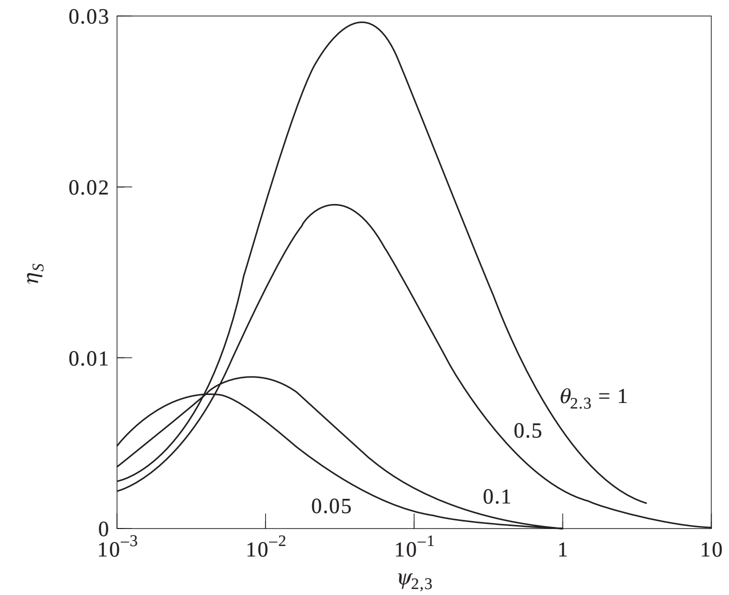

Figure 4. Variation of system loss factor η_s with $\psi_{2,3}$; $\theta_{1,3} = 0{\cdot}1$, $\eta_2 = 0{\cdot}1$, $\alpha_{1,3} = 1{\cdot}0$.

values of η_s and $k\eta_s$ obtained from these equations after including only transverse inertia terms, with those from equations (7–9), is given in Table 1. There is a large difference at higher values of G_2 and $n\pi/L$ if the shear effect in layers 1 and 3 is ignored.

Mead [23] carried out a detailed comparison of the equations for flexural vibrations of damped sandwich beams. A sixth order equation in transverse displacement w is obtained from [17, 18] which is similar to that obtained from equations (7–9). However, Yan and Dowell [24] obtained a fourth order equation in w for the same problem. The difference was attributed to the fact that in reference [24], an assumption of uniform shear stress through the depth of the whole sandwich was made, which is not strictly applicable especially for a soft and thin core. The solution of governing equations for damped

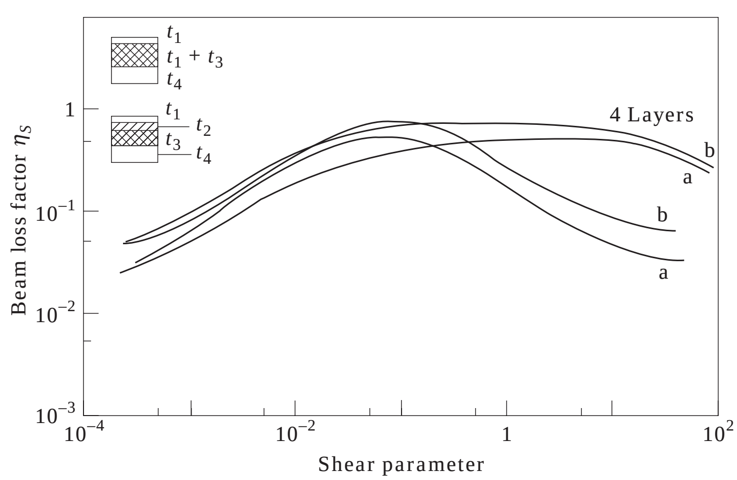

Figure 5. Comparison of three and four layer arrangements $\theta_{1,4} = \theta_{2,4} = 0{\cdot}5$, $\eta_2 = \eta_3 = \beta_3 = 1$, $\alpha_{3,4} = 0{\cdot}01$, $\alpha_{1,4} = 0{\cdot}33$. Curves a: $\theta_{2,3} = 2{\cdot}5$; curves b: $\theta_{3,4} = 5$. Three layers, $G_3/G_2 = 1000$.

TABLE 1

Influence of shear effect in layers 1 and 3

	G_2 (N/cm^2)					
	765		7650		76 500	
$n\pi/L$/cm	0·224	0·9	0·224	0·9	0·22 4	0·9
% difference in η_s	0·26	7·4	1·1	8·2	1·9	15·9
% difference in $k\eta_s$	1·4	21·1	2·4	22·4	4·1	35·4

sandwich beams for various boundary conditions is given in reference [25, 26]. In reference [27], an approximate solution for finding the forced response to harmonic excitation is given for fixed–fixed and cantilever type boundary conditions, using the Rayleigh–Ritz method and correspondence principle of viscoelasticity.

In a recent work [28], a refinement of the equations by taking variation of w across thickness and slippage at the interfaces has been done and is applicable to thick and flexible core materials. Not much work appears to have been done on the non-linear effects in stress–strain relations in the viscoelastic core and on the development of closed form simplified solutions for different boundary condition cases from various types of structural elements.

3.2. BEAMS WITH PARTIAL COVERAGE

Studies have been conducted on beams with constrained layer treatment which is not applied on the entire length of the beam. It was shown in reference [29] that a stiff viscoelastic layer gives a higher system loss factor for the case of a partially covered beam, compared to a fully covered one. Plunkett and Lee [30] have carried out an analysis for determining optimum length of the constraining layer, which may give a high value of maximum system damping. Markus [31] reported an analysis for finding the system loss factor for partially covered simply supported beams with constrained viscoelastic treatment, assuming the mode shape as that of the untreated beam. For the mode under consideration, the system loss factor was expressed as the ratio of energy dissipated per cycle to the maximum strain energy stored during the cycle of the harmonic motion.

Damping analysis of partially covered beams with constrained viscoelastic layer treatment, as in Figure 6(a), has been reported in reference [32]. Results of two formulations, based on approximate simplified methods viz. Markus's method and the Rayleigh–Ritz method, are compared with those obtained by exact solution of equations of motion and a reasonable agreement is seen. Results from reference [33] for various locations and percentage coverage are shown in Figures 6(b) and 6(c). It is seen from the former that a central coverage gives higher values of η_s for the first mode while the latter figure shows that for suitably chosen parameters, higher values of η_s may be obtained for a partially covered beam compared to that obtained for a fully covered one. This may not be true for some other values of chosen parameters like shear modulus G_2 of the viscoelastic material. The parameters chosen for Figure 6(b) are: $L = 1{\cdot}08$ m, $t_3 = 0{\cdot}005$ m, $\theta_{1,3} = 0{\cdot}1$, $\theta_{2,3} = 0{\cdot}5$, $\alpha_{1,3} = 1$, $E_3 = 2{\cdot}07 \times 10^{11}$ N/m^2, $\eta_2 = 0{\cdot}38$. The parameters chosen for Figure 6(c) are identical except that $G_2 = 2 \times 10^7$ N/m^2. In arrangement 1, the added layers start from the left end of the beam while for arrangement 2, the distance $a = (L - P_a)/4$. In arrangement 3, the added layers are symmetrical about the centre of the beam.

Some of the recent work includes determination of damping characteristics of partially covered five layered beams with alternate elastic and viscoelastic layers, using strain energy analysis [34]. Further, Mantenna *et al*. [35] have carried out experimental and analytical studies, the latter being based on FEM and modal strain energy technique, for the case of a composite material laminated beam, with constrained viscoelastic layer treatment.

3.3. ANALYSIS OF COMPLEX STRUCTURAL ELEMENTS AND STRUCTURES

The analysis of structural elements other than the beams with viscoelastic damping treatment has been carried out by several authors. The work for rectangular plates with damping treatment is reported in references [16, 33, 36, 37], for doubly curved panels in reference [38] and for cylindrical shells in references [39, 40]. Reference may be made to a recent publication [3] for additional references on the above topics.

3.3.1. *Finite element method*

Finite element methods have been developed by several researchers [41–44] for use on complex structures. The algebra involved is cumbersome and obtaining solutions is time-consuming in view of the large number of nodal displacements and number of elements needed, especially for analysis at high frequencies.

The effectiveness of damping treatment may be determined by the complex eigenvalue method or the modal strain energy (MSE) method. The MSE method [43] assumes that the real normal modes of the undamped system may be used for the damped structure and that the modal loss factor may be found from the summation of the product of material loss factor of each element and the fraction of elastic strain energy in that element. This

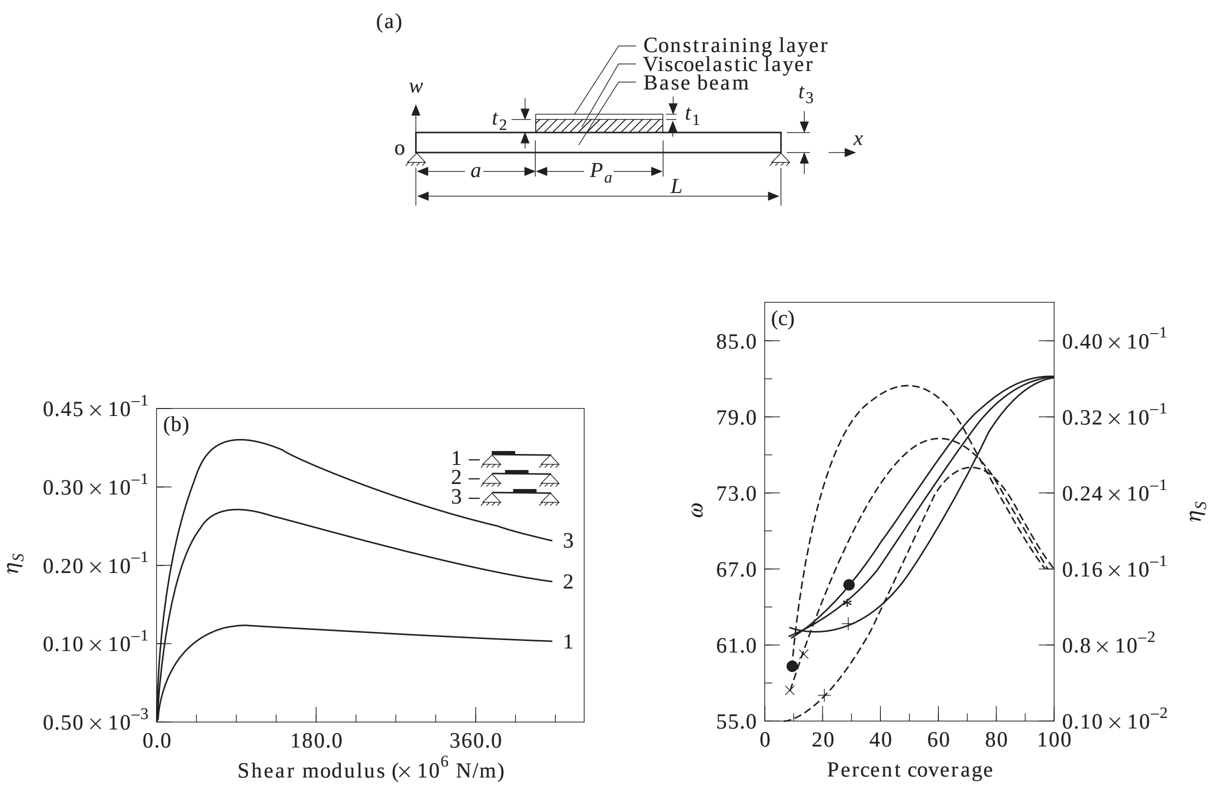

Figure 6. Partially covered beam characteristics. (a) Partially covered sandwich beam; (b) variation of η_s with core shear modulus; coverage—40%, mode 1; (c) resonance frequency and associated system loss factor with the coverage percentages; ——, ω; – – –, η_s; +, 1; ×, 2; ●, 3. Mode 1.

method has been found to be less time consuming compared to the complex eigenvalue method and is based on the approach earlier suggested by Ungar and Kerwin [45].

3.3.2. *Structural dynamic modification method*

Structural dynamic modification or re-analysis techniques are methods for efficient evaluation of modified dynamic characteristics of a complex system, using characteristics of the unmodified system. Chen *et al*. [46] have given a matrix perturbation method for vibration modal analysis. Shen and Stevens [47] used perturbation techniques for studying the influence of additive unconstrained damping treatment on the eigenvalues and system damping. The method is computationally efficient and useful where a number of damping configurations have to be tried for the purpose of design. In references [48, 49], this method has been applied to fully and partially covered constrained beams, with viscoelastic damping, and to complex structures of 'F' and inverted 'L' types, for determining eigenfrequencies and modal loss factors, with the additive treatment taken as a modification of the original untreated structure. The eigenvalues of the untreated system have been estimated by expressing the increments in eigen parameters in terms of increments in stiffness and mass matrices of the system, during modification.

The perturbation method has also been used for response re-analysis of both unconstrained and constrained types of viscoelastically damped beams and complicated structures, made up of beams [50, 51]. For the constrained types of viscoelastically damped beams, the original and modified matrices, are of different order and order reduction of matrices for the modified beams is achieved by Guyan's reduction [52].

3.4. SHOCK EXCITATION

Since complex modulus representation can only be used for VEMs for harmonic excitation, a differential operator form has been employed for shock and random excitations. A 4-element model shown in Figure 7(a) is seen to represent the characteristics of PVC till about 400 Hz [53]. Its shear modulus is represented by

$$G(\mathrm{D}) = (B + \gamma\mathrm{D} + \phi\mathrm{D}^2)/(1 + \alpha\mathrm{D}), \tag{10}$$

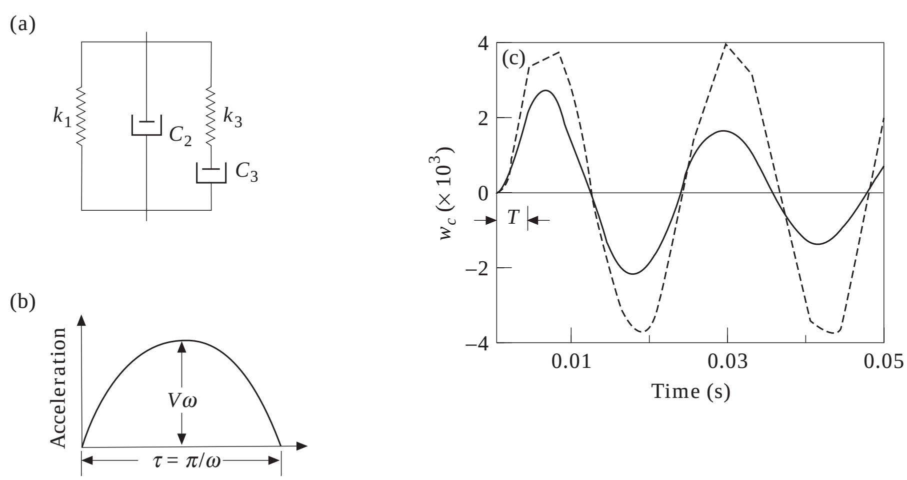

Figure 7. Response of damped and undamped beams to shock excitation ($\tau = \pi/1500$ s, $V = 20$ cm/s, L = 50 cm, b = 5 cm, $t_1/t_3 = 1$, $t_2/t_3 = 0{\cdot}5$, $\rho_1 = \rho_3 = 0{\cdot}28 \times 10^{-5}$ kgs²/cm⁴, $\rho_2 = \rho_3/2$, $E_1 = E_3 = 7 \times 10^6$ N/cm², $t_3 = 0{\cdot}25$ cm), (a) Four element viscoelastic model. (b) Half sine wave pulse acceleration. (c) Time history of shock response $\bar{w}_c$; – – –, undamped; ——, damped.

where

$$D = d/dt, \quad \alpha = c_3/k_3, \quad B = k_1, \quad \gamma = c_2 + c_3 + k_1 c_3/k_3, \quad \phi = c_2 c_3/k_3.$$

Typical results for $\bar{w}_c = w/L$ at the centre of a simply supported beam with viscoelastic core, with both ends subjected to a half sine pulse as in Figure 7(b), are given in Figure 7(c). The results [54] are compared with an identical undamped beam with shear modulus k_1, which may be treated as the static modulus of the VEM. The values of constants chosen for the viscoelastic material are: $k_1 = 350$ N/cm^2, $k_3 = 845$ N/cm^2, $c_2 = 0{\cdot}282$ N$_s$/cm^2, $c_3 = 0{\cdot}225$ N$_s$/cm^2. It is seen that the use of VEM, for the core, not only reduces the peak response but also results in decay of vibrations. Some of the studies concerning unconstrained and constrained type damped structures, subjected to shock excitation, are reported in reference [55, 56].

4. MULTI-PARAMETER OPTIMISATION STUDIES

In view of the large number of parameters involved in viscoelastically damped systems, it is desirable to carry out multi-parameter optimisation, with specified geometrical and physical constraints and to arrive at a dynamically optimum configuration.

Lunden [57] carried out optimisation studies to find optimum distribution of unconstrained distributed damping on beams and frames, with the objective of minimising resonant vibrations, subject to constraints on weight or cost of the additive damping, with constraints on the loss factor of the available material. An unconstrained minimisation function with exterior penalty terms is formulated and a sequential unconstrained minimisation technique to allow for constraints is used. It is seen that the redistribution of uniform additive damping in an optimum non-uniform way would reduce the response by 40 to 60% for the same weight or cost of additive treatment.

Lall *et al*. [58] have carried out multi-parameter optimum design studies for a sandwich plate with constrained viscoelastic core. The objective functions chosen were the modal system loss factor and displacement response, with design variables as the layer material densities, thicknesses and temperature. Linear relationship between material density and Young's modulus of Krokosky [59] were employed in addition to the temperature–frequency dependence principle [60], for simplification. In reference [61], optimisation studies were carried out in order to determine the optimum parameters of a four element model, used to represent the viscoelastic characteristics of the core for the sandwich plate. From the above, the in-phase shear modulus and loss factor of the viscoelastic material were plotted against frequency, giving the desired complex shear modulus variation for the material.

Results of optimisation studies [33] for optimum parameters of a partially covered plate of Figure 8, with constrained VEM, are compared in Table 2 for additive damping at locations 1 and 13 on the plate with a fully covered plate for the same total mass. F.C. refers to fully covered plate and PCL-1 refers to partially covered location 1. Similarly PCL-13 is for location 13 in the Figure. For the base plate, the parameters are $L = W = 0{\cdot}4$ m, thickness t_3 of the base plate $= 0{\cdot}005$ m, that of the viscoelastic layer $t_2 = 0{\cdot}0025$ m and that of the constraining layer $t_1 = 0{\cdot}0005$ m. For the VEM, the in-phase shear modulus $= 4 \times 10^6$ N/m^2 and the loss factor $= 0{\cdot}38$. The Young's modulus of the base and constraining layers $= 2{\cdot}07 \times 10^{11}$ N/m^2 and Poisson's ratio $= 0{\cdot}334$. The objective function was to maximize the system loss factor $\eta_{m,n}$ for the m, nth mode, with design parameters as P_L, P_W, t_1 and t_2, with the patch coverage area restricted to 40% of the base plate area and the added mass of the patch being equal to that for the fully covered case. It is seen from Table 2 that for the same total mass of the added layers, depending

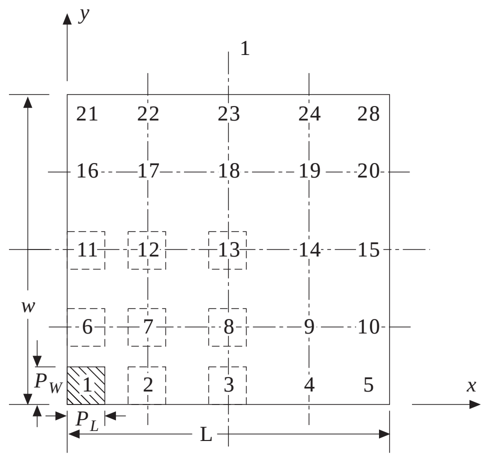

Figure 8. Various locations for additive damping on a plate.

on the mode under consideration, one may, by optimum choice of parameters, obtain a higher system loss factor $\eta_{m,n}$ for the partially covered plate than for a fully covered one [33].

Optimal constrained layer damping of beams, constrained by viscoelastic layers, has also been determined for sandwich beams with viscoelastic core [62], with layer thicknesses as design variables. Recently [63], studies have been reported for partially covered beams with constrained layer damping, taking nodal co-ordinates of the finite element model used as the design variables.

5. ROTOR SYSTEMS WITH VISCOELASTIC SUPPORTS

External viscoelastic damping at the supports of rotating systems, can be quite effective in controlling the dynamic response of the system [64]. Analysis of a Jeffcott rotor, with ball bearings at each end and with viscoelastic supports, showed that the imbalance response can be reduced considerably for an optimum value of stiffness and a high value of the material loss factor [65]. It has also been shown [66–68] that the stability limit of the rotor–shaft system, in case of instability due to material damping of the system or due to hydrodynamic bearings at the ends, can be improved by using suitable viscoelastic support parameters. The stability analysis has been carried out, using the operator form of the constitutive equations, corresponding to a four element type of model of the viscoelastic material.

The results of a simple algorithm [69, 70] to get desired characteristics of viscoelastic supports of a simple Jeffcott rotor are shown in Figure 9. The support stiffness parameter $\beta_1 = K/K_s$, where K is in-phase stiffness of support, with loss factor η, and K_s is the shaft stiffness at the disc location. The parameters for Figure 9 are $M_1/M_2 = 0{\cdot}2$, $K_b/K_s = 2$; M_1 is the support mass, M_2 being the disc mass and K_b is the bearing stiffness; $\delta_R = \omega/\omega_n$, ω being the angular speed of the rotor and ω_n is the undamped natural frequency of the rotor–shaft system, having rigid supports. R_D is the ratio of amplitude of vibration of disc to the eccentricity of the disc.

The algorithm is based on the location of the two resonance frequencies δ_{R1} and δ_{R2} as in Figure 9(b). For each value of δ_R, values of β_1 and η are tried so that δ_{R3} and δ_{R4} are $\geqslant c$, the comparison constant, which is suitably chosen in the algorithm so that the two resonant frequencies are located as above. The other possibilities viz. (1) $\delta_{R3} < c$ and

TABLE 2

Comparison of optimum values of parameters for fully and partially covered plates

	Mode								
	$m = 1$, $n = 1$			$m = 1$, $n = 2$			$m = 2$, $n = 2$		
Values	FC	PCL-1	PCL-13	FC	PCL-1	PCL-13	FC	PCL-1	PCL-13
ω_{mn}	940·6	897·3	897·1	2260·0	2310·3	2247·2	3578·3	3660·5	3730·1
η_{mn}	0·048	0·024	0·043	0·021	0·0251	0·035	0·0134	0·018	0·0175
P_L (m)	0·4	0·24	0·19	0·4	0·26	0·17	0·4	0·2	0·36
P_W (m)	0·4	0·3	0·35	0·4	0·28	0·39	0·4	0·36	0·17
t_1 (m)	$0{\cdot}5 \times 10^{-3}$	$0{\cdot}23 \times 10^{-2}$	$0{\cdot}26 \times 10^{-3}$	$0{\cdot}5 \times 10^{-3}$	$0{\cdot}24 \times 10^{-2}$	$0{\cdot}26 \times 10^{-2}$	$0{\cdot}5 \times 10^{-3}$	$0{\cdot}25 \times 10^{-2}$	$0{\cdot}27 \times 10^{-2}$
t_2 (m)	$0{\cdot}25 \times 10^{-2}$	$0{\cdot}12 \times 10^{-2}$	$0{\cdot}72 \times 10^{-3}$	$0{\cdot}25 \times 10^{-3}$	$0{\cdot}65 \times 10^{-2}$	$0{\cdot}5 \times 10^{-3}$	$0{\cdot}25 \times 10^{-2}$	$0{\cdot}5 \times 10^{-3}$	$0{\cdot}5 \times 10^{-3}$

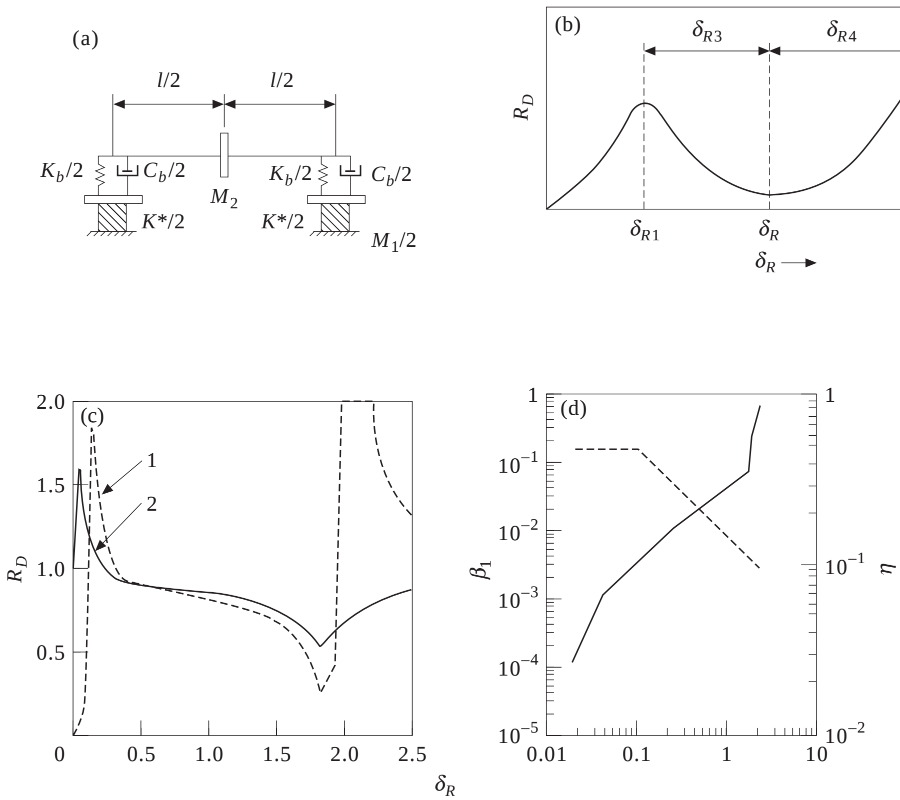

Figure 9. Results for a rotor with viscoelastic supports. (a) Viscoelastic supports; (b) typical imbalance response; (c) imbalance response of rotor for frequency independent and dependent parameters; (d) frequency dependent support parameters; – – –, η; ——, β_1.

$\delta_{R4} > c$, (2) $\delta_{R3} > c$ and $\delta_{R4} < c$, and (3) $\delta_{R3} < c$ and $\delta_{R4} < c$, are discarded. The imbalance response of a frequency-dependent VEM (with characteristics as in Figure 9(d)) is seen to be lower than that of a frequency-independent VEM at the supports, as shown in Figure 9(c). For the latter case $\beta_1 = 0{\cdot}02$, $\eta = 0{\cdot}5$. Depending on the initial chosen values of β_1 and η, different characteristics of the support material may be obtained.

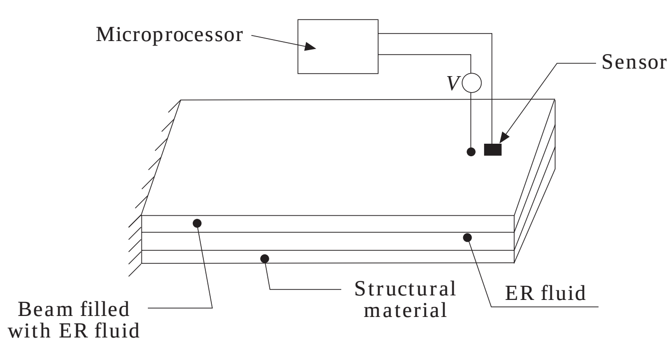

Figure 10. Composites with electro-rheological materials.

6. USE OF INTELLIGENT TREATMENTS

Some of the recent developments involve the use of intelligent treatments whose damping characteristics can change in response to the applied voltage. Figure 10 shows an arrangement using electro-rheological (ER) materials which are currently being explored. Stiffness and damping properties of ER materials can be controlled by voltage V applied through feedback devices. The structures of this type are called ultra-advanced intelligent composite materials [71].

Another novel arrangement uses active piezoelectric layers on both sides of a viscoelastic core which are attached to the base layer. One of the active layer acts as a sensor whose output after amplification is applied to the active constrained layer, giving added damping due to increased shear strain in the core [72]. Such damping treatments are called intelligent constrained layer (ICL) or active constrained layer (ACL) treatments in which constrained layer damping and active feedback control are integrated. Shen [73] has recently studied the control characteristics of such an arrangement.

7. EMERGING TRENDS AND FUTURE WORK

These are expected to be as follows:

(1) There will be greater emphasis on built-in damping at the design stage rather than providing it as an afterthought, thus treating damping as a design parameter [74].

(2) Keeping in view the reliability and economic considerations, a combination of passive damping and active devices for continuous flexible systems, may be used.

(3) Further attention may be devoted to analysis for refinement of theories to incorporate non-linear viscoelastic models, shock and random excitations, development of approximate closed form solutions for structural elements with additive viscoelastic damping and analysis of machinery isolation systems incorporating high damping VEMs.

(4) Work on development of viscoelastic materials, with less variation of dynamic properties, over a wide range of temperatures and frequencies, in addition to work on development of intelligent materials giving desired dynamic stiffness and damping over prescribed frequency and temperature ranges and analysis of structures with the above, is likely to be emphasised.

(5) Optimisation studies of the dynamic response, covering a large frequency range over a number of modes, need to be carried out. Dynamic optimum design studies on complex systems involving sensitivity analysis and re-analysis should also be of interest.

(6) Analytical evaluation of reduction in noise radiation from structures with additive damping and exposed to a dynamic environment does not appear to have been carried out and should be of interest.

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APPENDIX A: SHEAR EFFECT CONSIDERED IN LAYERS 1 AND 3

$$\frac{E_1}{2}\left(\tfrac{2}{3}\,t_1^3\bar{u}_1'' - t_1^2 u_2'' + \frac{t_1^2 t_2}{2}\,\bar{u}_2''\right) - k_1\,G_1\,(t_1\,\bar{u}_1 + t_1\,w') = \frac{\rho_1}{2}\left[\ddot{\bar{u}}_1\,\frac{t_1^3}{2} - \ddot{u}_2\,t_1^2 + \frac{t_1^2 t_2}{2}\,\ddot{\bar{u}}_2 + \frac{t_1^3}{6}\,\ddot{\bar{u}}_1\right], \tag{A.1}$$

$$\frac{E_1}{2}\,(2t_1\,u_2'' - t_1\,t_2\,\bar{u}_2'' - t_1^2\bar{u}_1'') + \frac{E_3}{2}\,(2t_3\,u_2'' + t_2\,t_3\,\bar{u}_2'' + t_3^2\bar{u}_3'' + \frac{E_2}{2}\,(2u_2''\,t_2)$$
$$= \frac{\rho_1}{2}\,[2t_1\,\ddot{u}_2 - t_1\,t_2\,\ddot{\bar{u}}_2 - t_1^2\ddot{\bar{u}}_1] + \frac{\rho_3}{2}\,[2t_3\,\ddot{u}_2 + t_3\,t_2\,\ddot{\bar{u}}_2 + t_3^2\ddot{\bar{u}}_3] + \frac{\rho_2}{2}\,[2t_2\,\ddot{u}_2], \tag{A.2}$$

$$\frac{E_1}{2}\left(\frac{t_1\,t_2^2}{2}\,\bar{u}_2'' - t_1\,t_2\,u_2'' + \frac{t_1^2 t_2}{2}\,\bar{u}_1''\right) + \frac{E_3}{2}\left(\frac{t_3\,t_2^2}{2}\,\bar{u}_2'' - t_2\,t_3\,u_2'' + \frac{t_3^2 t_2}{2}\,\bar{u}_2''\right)$$
$$+ \frac{E_2}{2}\left(\frac{t_2^3}{6}\,\bar{u}_2''\right) - k_2\,G_2\,t_2\,(w_2' + \bar{u}_2)$$
$$= \frac{\rho_1}{2}\left[\frac{t_1\,t_2^2}{2}\,\ddot{\bar{u}}_2 - t_1\,t_2\,\ddot{u}_2 + \frac{t_1^2 t_2}{2}\,\ddot{\bar{u}}_1\right] + \frac{\rho_3}{2}\left[\frac{t_3\,t_2^2}{2}\,\ddot{\bar{u}}_2 + t_3\,t_2\,\ddot{u}_2 + \frac{t_2\,t_3^2}{2}\,\ddot{\bar{u}}_3\right] + \frac{\rho_2}{2}\left[\ddot{\bar{u}}_2\,\frac{t_2^3}{6}\right], \tag{A.3}$$

$$\frac{E_3}{2}\left[\tfrac{2}{3}\,t_3^3\bar{u}_3'' + t_3^2 u_2'' + \frac{t_2\,t_3^2}{2}\,\bar{u}_2''\right] - k_3\,G_3\,t_3\,(\bar{u}_3 + w_2') = \frac{\rho_3}{2}\left[\tfrac{2}{3}\,t_3^3\ddot{\bar{u}}_3 + t_3^2\ddot{u}_2 + \frac{t_2\,t_3^2}{2}\,\ddot{\bar{u}}_2\right], \tag{A.4}$$

$$k_1\,G_1\,t_1\,(\bar{u}_1' + w'') + k_3\,G_3\,t_3\,(\bar{u}_3' + w'') + k_2\,G_2\,t_2\,(\bar{u}_2' + w'')$$
$$= (\rho_1\,t_1 + \rho_2\,t_2 + \rho_3\,t_3)\ddot{w} - f(x)\sin\omega t. \tag{A.5}$$

Boundary conditions for simply supported beam are

$$\bar{u}_1' = \bar{u}_2' = \bar{u}_3' = u_2' = 0, \qquad w = 0. \tag{A.6}$$